

HOMEWORK 13
LINEAR ALGEBRA
(MATH 442), SPRING, 2026

DUE: 05/15/2026 (OPTIONAL)

SECTION 8.D - TRACE

Problem 1. Suppose $P \in \mathcal{L}(V)$ satisfies $P^2 = P$. Prove that $\text{trace } P = \dim \text{range } P$.

Problem 2. Suppose V is a finite-dimensional inner product space and $T \in \mathcal{L}(V)$. Prove that $\text{trace } T^* = \overline{\text{trace } T}$.

SECTION 9.D - DETERMINANT

Problem 3. Suppose $T \in \mathcal{L}(V)$. Define $g : \mathbb{F} \rightarrow \mathbb{F}$ by $g(x) = \det(I + xT)$. Show that $g'(0) = \text{trace } T$.

Problem 4. Suppose V is an inner product space, $e_1, \dots, e_n$ is an orthonormal basis of V , and $T \in \mathcal{L}(V)$ is a positive operator. Prove that

$$\det T \leq \prod_{k=1}^n \langle Te_k, e_k \rangle.$$