

HOMework 11

LINEAR ALGEBRA

(MATH 442), SPRING, 2026

DUE: 05/01/2026

SECTION 7.B - SPECTRAL THEOREM

Problem 1. Prove that a normal operator on a complex inner product space is self-adjoint if and only if all its eigenvalues are real.

Problem 2. Suppose V is a complex inner product space and $T \in \mathcal{L}(V)$. Prove that T is normal if and only if there exists a polynomial $p \in \mathcal{P}(\mathbb{C})$ such that $T^* = p(T)$.

SECTION 7.C - POSITIVE OPERATORS

Problem 3. Suppose T is a positive operator on V and $v, w \in V$ are such that $Tv = w$ and $Tw = v$. Prove that $v = w$.

Problem 4. Suppose T is a positive operator on V . Prove that $\text{null } \sqrt{T} = \text{null } T$ and $\text{range } \sqrt{T} = \text{range } T$.