Homework 10
Linear Algebra
(MATH 442), Spring, 2026

Due: 04/17/2026

Section 6.C - Orthogonal Complements and Minimization Problems

Problem 1. Suppose U is a subspace of an inner product space V. Show that $P_{U^\perp} = I - P_U$.

Problem 2. Suppose $T \in \mathcal{L}(V)$ is such that $T^2 = T$ and every vector in null T is orthogonal to every vector in range T. Prove that there exists a subspace U of V such that $T = P_U$.

Problem 3. Find $p \in \mathcal{P}_3(\mathbb{R})$ such that $p(0) = 0$, $p'(0) = 0$, and $\int_0^1 (2 + 3x - p(x))^2 dx$ is as small as possible.

Section 7.A - Self-Adjoint and Normal Operators

Problem 4. Suppose $\dim(V) < \infty$, $T \in \mathcal{L}(V)$, and $\lambda \in \mathbb{F}$. Prove that λ is an eigenvalue of T if and only if $\overline{\lambda}$ is an eigenvalue of T^*. (Hint: It may be easier to prove the equivalent statement that λ is not an eigenvalue of T if and only if $\overline{\lambda}$ is not an eigenvalue of T^*).

Problem 5. Prove that the product of two self-adjoint operators on V is self-adjoint if and only if the two operators commute.