Homework 9
Linear Algebra
(MATH 442), Spring, 2026

Due: 04/10/2026

Section 6.A - Inner Products and Norms

Problem 1. Suppose V is a real inner product space. Show that if $v, w \in V$ have the same norm, then $v + w$ is orthogonal to $v - w$.

Problem 2. Suppose V is an inner product space and $v \in V$ is a nonzero vector. Prove that if $w \in V$ and $\|w\| = 1$, then

$$\left\| v - \frac{v}{\|v\|} \right\| \leq \|v - w\|$$

with equality only when $w = \dfrac{v}{\|v\|}$. That is, prove that $\dfrac{v}{\|v\|}$ is the unique closest element to v on the unit sphere of V (i.e. the set $\{v \in V : \|v\| = 1\}$).

Section 6.B - Orthonormal Bases

Problem 3. Suppose $\langle \cdot, \cdot \rangle_1$ and $\langle \cdot, \cdot \rangle_2$ are two inner products on a vector space V such that $\langle v, w \rangle_1 = 0$ if and only if $\langle v, w \rangle_2 = 0$. Prove that there is a positive number c such that $\langle v, w \rangle_1 = c\langle v, w \rangle_2$ for every $v, w \in V$.

Problem 4. Let V be a finite-dimensional inner product space and suppose $v_1, \ldots, v_n$ is a basis for V. Prove that there exists a basis $u_1, \ldots, u_n$ of V such that

$$\langle v_j, u_k \rangle = \begin{cases} 0 & j \neq k \\ 1 & j = k \end{cases}$$