Homework 8
Linear Algebra
(MATH 442), Spring, 2026

Due: 04/01/2026

Section 5.D - Diagonalizable Operators

Problem 1. Suppose $T \in \mathcal{L}(V)$. Let $\lambda_1, \dots, \lambda_m$ denote the distinct nonzero eigenvalues of T. Prove that

$$\dim E(\lambda_1, T) + \ldots + \dim E(\lambda_m, T) \leq \dim \text{range } T.$$

Problem 2. Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix with respect to some basis for V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of this matrix precisely $\dim E(\lambda, T)$ times.

Section 5.E - Commuting Operators

Problem 3. Prove that a pair of operators on a finite-dimensional vector space commute if and only if their dual operators commute.

Problem 4. Give an example of two commuting operators S, T on a finite-dimensional real vector space such that $S + T$ has a eigenvalue that does not equal an eigenvalue of S plus an eigenvalue of T and such that ST has a eigenvalue that does not equal an eigenvalue of S times an eigenvalue of T.