Homework 7
Linear Algebra
(MATH 442), Spring, 2026

Due: 03/27/2026

Section 5.B (Part 2) - Eigenvalues and the Minimal Polynomial

Problem 1. Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$ is invertible. Prove that the exists a polynomial $p \in \mathcal{P}(\mathbb{F})$ such that $\deg p \leq \dim V - 1$ and $T^{-1} = p(T)$.

Problem 2. Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Prove that the minimal polynomial of $T' \in \mathcal{L}(V')$ is equal to the minimal polynomial of T.

Section 5.C - Upper-Triangular Matrices

Problem 3. Suppose $T \in \mathcal{L}(V)$ and $v_1, \ldots, v_n$ is a basis of V with respect to which the matrix of T is upper-triangular.

1. Show that for every positive integer m, the matrix of T^m is also upper-triangular with respect to the basis $v_1, \ldots, v_n$.
2. Suppose additionally that T is invertible. Show that the matrix of T^{-1} is also upper-triangular with respect to the basis $v_1, \ldots, v_n$.

Problem 4. Prove or give a counterexample: If $T \in \mathcal{L}(V)$ is such that T^2 has an upper-triangular matrix with respect to some basis of V, then T has an upper-triangular matrix with respect to some basis of V.