Homework 4
Linear Algebra
(MATH 442), Spring, 2026

Due: 02/20/2026

Section 3.C - Matrices

Problem 1. Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Show that with respect to each choice of bases of V and W, the matrix of T has at least $\dim\big(\operatorname{range}(T)\big)$ nonzero entries.

Problem 2. Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Prove that $\dim\big(\operatorname{range}(T)\big) = 1$ if and only if there exist a basis of V and a basis of W such that with respect to these bases, all entries of $\mathcal{M}(T)$ equal 1.

Problem 3. Suppose that $D \in \mathcal{L}\big(\mathcal{P}_3(\mathbb{R}), \mathcal{P}_2(\mathbb{R})\big)$ is the differentiation map defined by $Dp = p'$. Find a basis of $\mathcal{P}_3(\mathbb{R})$ and $\mathcal{P}_2(\mathbb{R})$ such that the matrix of D with respect to these bases is

$$\mathcal{M}(D) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Section 3.D - Invertibility and Isomorphic Vector Spaces

Problem 4. Suppose V is finite-dimensional, U is a subspace of V, and $S \in \mathcal{L}(U, V)$. Prove there exists an invertible linear map $T \in \mathcal{L}(V)$ such that $Tu = Su$ for every $u \in U$ if and only if S is injective.

Problem 5. Suppose V is finite-dimensional and $T \in \mathcal{L}(V, W)$ is surjective. Prove that there exists a subspace $U \leq V$ such that $T\big|_U$ is an isomorphism.
(Here, $T\big|_U$ refers to the restriction of T to U, i.e. the map $T\big|_U \in \mathcal{L}(U, W)$ such that $T\big|_U u = Tu$ for all $u \in U$).