Homework 2
Linear Algebra
(MATH 442), Spring, 2026

Due: 02/06/2026

Section 1.C - Subspaces

Problem 1. Is the operation of addition on the subspaces of a vector space V commutative? That is, if U and W are subspaces of V, is it true that $U + W = W + U$?

Problem 2. Prove that the intersection of any collection of subspaces of V is a subspace of V.

Section 2.A - Span and Linear Independence

Problem 3. Suppose $v_1, \dots, v_m$ is linearly independent in V and let $w \in V$. Prove that if $v_1 + w, \dots, v_m + w$ is linearly dependent, then $w \in \operatorname{span}\left\{v_1, \dots, v_m\right\}$.

Problem 4. Prove or give a counterexample: If $v_1, \dots, v_m$ is linearly independent in V and $0 \neq \lambda \in \mathbb{F}$, then $\lambda v_1, \dots, \lambda v_m$ is linearly independent in V.

Section 2.B - Bases

Problem 5. Prove or give a counterexample: If v_1, v_2, v_3, v_4 is a basis of V and $U \leq V$ is such that $v_1, v_2 \in U$ and $v_3, v_4 \notin U$, then v_1, v_2 is a basis of U.

Problem 6. Suppose $U, W \leq V$ and $V = U \oplus W$. Prove that if $u_1, \dots, u_m$ is a basis of U and $w_1, \dots, w_n$ is a basis of W, then $u_1, \dots, u_m, w_1, \dots, w_n$ is a basis of V.