

7B Spectral Theorem

Real Spectral Theorem

7.26 invertible quadratic expressions

Suppose $T \in \mathcal{L}(V)$ is self-adjoint and $b, c \in \mathbf{R}$ are such that $b^2 < 4c$. Then

$$T^2 + bT + cI$$

is an invertible operator.

7.27 *minimal polynomial of self-adjoint operator*

Suppose $T \in \mathcal{L}(V)$ is self-adjoint. Then the minimal polynomial of T equals $(z - \lambda_1) \cdots (z - \lambda_m)$ for some $\lambda_1, \dots, \lambda_m \in \mathbf{R}$.

7.29 *real spectral theorem*

Suppose $\mathbf{F} = \mathbf{R}$ and $T \in \mathcal{L}(V)$. Then the following are equivalent.

- (a) T is self-adjoint.
- (b) T has a diagonal matrix with respect to some orthonormal basis of V .
- (c) V has an orthonormal basis consisting of eigenvectors of T .

Complex Spectral Theorem

7.31 *complex spectral theorem*

Suppose $\mathbf{F} = \mathbf{C}$ and $T \in \mathcal{L}(V)$. Then the following are equivalent.

- (a) T is normal.
- (b) T has a diagonal matrix with respect to some orthonormal basis of V .
- (c) V has an orthonormal basis consisting of eigenvectors of T .