

2B Bases

2.26 definition: *basis*

A *basis* of V is a list of vectors in V that is linearly independent and spans V .

2.28 *criterion for basis*

A list $v_1, \dots, v_n$ of vectors in V is a basis of V if and only if every $v \in V$ can be written uniquely in the form

$$2.29 \quad v = a_1 v_1 + \dots + a_n v_n,$$

where $a_1, \dots, a_n \in \mathbf{F}$.

2.30 *every spanning list contains a basis*

Every spanning list in a vector space can be reduced to a basis of the vector space.

2.31 *basis of finite-dimensional vector space*

Every finite-dimensional vector space has a basis.

2.32 *every linearly independent list extends to a basis*

Every linearly independent list of vectors in a finite-dimensional vector space can be extended to a basis of the vector space.

2.33 *every subspace of V is part of a direct sum equal to V*

Suppose V is finite-dimensional and U is a subspace of V . Then there is a subspace W of V such that $V = U \oplus W$.