8D Trace: A Connection Between Matrices and Operators

8.47 definition: *trace of a matrix*

Suppose A is a square matrix with entries in $\mathbf{F}$. The *trace* of A, denoted by $\operatorname{tr} A$, is defined to be the sum of the diagonal entries of A.

8.49 *trace of AB equals trace of BA*

Suppose A is an m-by-n matrix and B is an n-by-m matrix. Then

$$\operatorname{tr}(AB) = \operatorname{tr}(BA).$$

8.50 *trace of matrix of operator does not depend on basis*

Suppose $T \in \mathcal{L}(V)$. Suppose $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ are bases of V. Then

$$\operatorname{tr} \mathcal{M}(T, (u_1, \ldots, u_n)) = \operatorname{tr} \mathcal{M}(T, (v_1, \ldots, v_n)).$$

8.51 definition: *trace of an operator*

Suppose $T \in \mathcal{L}(V)$. The *trace* of T, denote $\operatorname{tr} T$, is defined by

$$\operatorname{tr} T = \operatorname{tr} \mathcal{M}(T, (v_1, \ldots, v_n)),$$

where $v_1, \ldots, v_n$ is any basis of V.

8.52 *on complex vector spaces, trace equals sum of eigenvalues*

Suppose $\mathbf{F} = \mathbf{C}$ and $T \in \mathcal{L}(V)$. Then $\operatorname{tr} T$ equals the sum of the eigenvalues of T, with each eigenvalue included as many times as its multiplicity.

8.54 *trace and characteristic polynomial*

Suppose $\mathbf{F} = \mathbf{C}$ and $T \in \mathcal{L}(V)$. Let $n = \dim V$. Then $\operatorname{tr} T$ equals the negative of the coefficient of z^{n-1} in the characteristic polynomial of T.

9C Determinants

Defining the Determinant