

5D Diagonalizable Operators

5.48 definition: *diagonal matrix*

A *diagonal matrix* is a square matrix that is 0 everywhere except possibly on the diagonal.

5.50 definition: *diagonalizable*

An operator on V is called *diagonalizable* if the operator has a diagonal matrix with respect to some basis of V .

5.52 definition: *eigenspace*, $E(\lambda, T)$

Suppose $T \in \mathcal{L}(V)$ and $\lambda \in \mathbf{F}$. The *eigenspace* of T corresponding to λ is the subspace $E(\lambda, T)$ of V defined by

$$E(\lambda, T) = \text{null}(T - \lambda I) = \{v \in V : Tv = \lambda v\}.$$

Hence $E(\lambda, T)$ is the set of all eigenvectors of T corresponding to λ , along with the 0 vector.

5.54 *sum of eigenspaces is a direct sum*

Suppose $T \in \mathcal{L}(V)$ and $\lambda_1, \dots, \lambda_m$ are distinct eigenvalues of T . Then

$$E(\lambda_1, T) + \dots + E(\lambda_m, T)$$

is a direct sum. Furthermore, if V is finite-dimensional, then

$$\dim E(\lambda_1, T) + \dots + \dim E(\lambda_m, T) \leq \dim V.$$

5.55 *conditions equivalent to diagonalizability*

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Let $\lambda_1, \dots, \lambda_m$ denote the distinct eigenvalues of T . Then the following are equivalent.

- (a) T is diagonalizable.
- (b) V has a basis consisting of eigenvectors of T .
- (c) $V = E(\lambda_1, T) \oplus \dots \oplus E(\lambda_m, T)$.
- (d) $\dim V = \dim E(\lambda_1, T) + \dots + \dim E(\lambda_m, T)$.

5.58 *enough eigenvalues implies diagonalizability*

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$ has $\dim V$ distinct eigenvalues. Then T is diagonalizable.

