5.29 *$q(T) = 0 \iff q$ is a polynomial multiple of the minimal polynomial*

Suppose V is finite-dimensional, $T \in \mathcal{L}(V)$, and $q \in \mathcal{P}(\mathbf{F})$. Then $q(T) = 0$ if and only if q is a polynomial multiple of the minimal polynomial of T.

5.31 *minimal polynomial of a restriction operator*

Suppose V is finite-dimensional, $T \in \mathcal{L}(V)$, and U is a subspace of V that is invariant under T. Then the minimal polynomial of T is a polynomial multiple of the minimal polynomial of $T|_U$.

5.32 *T not invertible $\iff$ constant term of minimal polynomial of T is 0*

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Then T is not invertible if and only if the constant term of the minimal polynomial of T is 0.

Eigenvalues on Odd-Dimensional Real Vector Spaces

5.33 *even-dimensional null space*

Suppose $\mathbf{F} = \mathbf{R}$ and V is finite-dimensional. Suppose also that $T \in \mathcal{L}(V)$ and $b, c \in \mathbf{R}$ with $b^2 < 4c$. Then $\dim \operatorname{null}(T^2 + bT + cI)$ is an even number.

5.34 *operators on odd-dimensional vector spaces have eigenvalues*

Every operator on an odd-dimensional vector space has an eigenvalue.