5B The Minimal Polynomial

5.19 *existence of eigenvalues*

Every operator on a finite-dimensional nonzero complex vector space has an eigenvalue.

Eigenvalues and the Minimal Polynomial

5.21 definition: *monic polynomial*

A *monic polynomial* is a polynomial whose highest-degree coefficient equals 1.

5.22 *existence, uniqueness, and degree of minimal polynomial*

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Then there is a unique monic polynomial $p \in \mathcal{P}(\mathbf{F})$ of smallest degree such that $p(T) = 0$. Furthermore, $\deg p \le \dim V$.

5.24 definition: ***minimal polynomial***

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Then the *minimal polynomial* of T is the unique monic polynomial $p \in \mathcal{P}(\mathbf{F})$ of smallest degree such that $p(T) = 0$.

5.27 ***eigenvalues are the zeros of the minimal polynomial***

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$.

(a) The zeros of the minimal polynomial of T are the eigenvalues of T.

(b) If V is a complex vector space, then the minimal polynomial of T has the form

$$(z - \lambda_1) \cdots (z - \lambda_m),$$

where $\lambda_1, \ldots, \lambda_m$ is a list of all eigenvalues of T, possibly with repetitions.