

3.125 *dimension of the annihilator*

Suppose V is finite-dimensional and U is a subspace of V . Then

$$\dim U^0 = \dim V - \dim U.$$

3.127 *condition for the annihilator to equal $\{0\}$ or the whole space*

Suppose V is finite-dimensional and U is a subspace of V . Then

(a) $U^0 = \{0\} \iff U = V;$

(b) $U^0 = V' \iff U = \{0\}.$

3.128 *the null space of T'*

Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Then

- (a) $\text{null } T' = (\text{range } T)^0$;
- (b) $\dim \text{null } T' = \dim \text{null } T + \dim W - \dim V$.

3.129 *T surjective is equivalent to T' injective*

Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Then

$$T \text{ is surjective} \iff T' \text{ is injective.}$$

3.130 *the range of T'*

Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Then

(a) $\dim \text{range } T' = \dim \text{range } T$;

(b) $\text{range } T' = (\text{null } T)^0$.

3.131 *T injective is equivalent to T' surjective*

Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Then

T is injective $\iff T'$ is surjective.

Matrix of Dual of Linear Map

3.132 *matrix of T' is transpose of matrix of T*

Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Then

$$\mathcal{M}(T') = (\mathcal{M}(T))^t.$$

3.133 *column rank equals row rank*

Suppose $A \in \mathbf{F}^{m,n}$. Then the column rank of A equals the row rank of A .