HOMEWORK 4

CHAPTER 4: RELATIONS

4.1. Determine whether or not each of the following relations is reflexive, symmetric, antisymmetric, and/or transitive. Are any of these equivalence relations?

(i) The relation $\leq$ on $\mathbb{R}$.

(ii) Equality on $\mathbb{R}$, i.e. the set of ordered pairs of real numbers whose first and second coordinates are equal.

(iii) The relation $\mid$ on $\mathbb{N}$, where $a \mid b$ means that $b = an$ for some $n \in \mathbb{N}$.

(iv) The relation $\sim$ on $\mathbb{N}$ defined by $a \sim b$ if there is an integer $n > 1$ that evenly divides both a and b.

4.2. Define the relation $\equiv_5$ on $\mathbb{Z}$ by: $a \equiv_5 b$ if there is an integer n so that $b - a = 5n$. Show that $\equiv_5$ is an equivalence relation on $\mathbb{Z}$. Note: this is a fairly common equivalence relation. The phrase $a \equiv_5 b$ is usually read "a is equivalent to b modulo 5."

4.3. A relation on a set X is said to be a *partial order* on X if it is reflexive, antisymmetric, and transitive. Let $X = \mathcal{P}(\mathbb{R})$ and consider the inclusion relation defined by $A \subset B$.

(i) Show that $\subset$ is a partial order on X.

(ii) Is strict inclusion $\subsetneq$ a partial order on X?

4.4. Define a relation $\sim$ on $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ by $(x, y) \sim (w, z)$ if $x^2 + y^2 = w^2 + z^2$. Show that $\sim$ is an equivalence relation on $\mathbb{R}^2$.

4.5. Consider the equivalence relation defined in Exercise 4.4. For each point $(x, y) \in \mathbb{R}^2$, the equivalence class $[(x, y)]$ is a familiar geometric figure in $\mathbb{R}^2$. What is it?