HOMEWORK 3

CHAPTER 3: SET THEORY

3.1. Here are some common infinite sets:
$P = \{n \mid n \text{ is prime}\}$ is the set of all prime numbers.
$E = \{n \mid n = 2k \text{ for some } k \in \mathbb{Z}\}$ is the set of all even integers.

How would you write the set of all odd integers in set builder notation? What about the set of all integer powers of 2?

3.2. Let $A = \{1, 2, \{1, 2\}, \{1, 3\}, 4\}$. Determine whether each of the following is en element of A, a subset of A, both an element of A and a subset of A, or neither an element of A nor a subset of A.

(i) 1

(ii) 3

3.3. Let A and B be sets and suppose you know that A is not a subset of B. Which of the following is necessarily true? Choose all correct responses.

(i) If $x \in A$, then $x \notin B$.

(ii) If $x \in B$, then $x \in A$.

(iii) There is an element $x \in A$ so that $x \notin B$.

(iv) There is an element $x \in B$ so that $x \notin A$.

3.4. Let A and B be sets and suppose that $x \notin A \cup B$. Which of the following is necessarily true? Choose all correct responses.

(i) $x \notin A$ or $x \notin B$

(ii) $x \notin A$ and $x \notin B$

(iii) $x \in A$ or $x \in B$

(iv) $x \in A$ and $x \in B$

3.5. Let A and B be sets and suppose that $x \notin A \cap B$. Which of the following is necessarily true? Choose all correct responses.

(i) $x \notin A$ or $x \notin B$

(ii) $x \notin A$ and $x \notin B$

(iii) $x \in A$ or $x \in B$

(iv) $x \in A$ and $x \in B$

3.6. Define the following sets: $A = \{1,3,9,27\}$, $B = \{1,2,4,8\}$, $P = \{n \mid n \text{ is a prime integer}\}$, and $E = \{n \mid n = 2k \text{ for some } k \in \mathbb{N}\}$. Find each of the following:

(i) $A \cup B$

(ii) $A \cap B$

(iii) $P \cap E$

3.7. For any sets A and B, prove that:

(i) $A \subset A \cup B$

(ii) $A \cap B \subset A$

(iii) $A \cap \emptyset = \emptyset$

3.8. Let A, B, and C be sets. Prove that $A \subset B \cap C$ if and only if $A \subset B$ and $A \subset C$.

3.9. Let A, B, and C be sets. Prove that if $A \subset B \cup C$ and $A \cap B = \emptyset$, then $A \subset C$.

3.10. Let A and B be sets in a universal set U. Prove the following:

(i) $A \setminus B = A \cap B'$

(ii) $A \setminus B = A$ if and only if $A \cap B = \emptyset$.

(iii) $A \setminus B = \emptyset$ if and only if $A \subset B$.

3.11. Determine whether or not each of the following is true. If so, provide a proof. If not, provide a counterexample.

(i) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

(ii) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

3.12. Let A and B be sets.

(i) Show that $\mathcal{P}(A) \cup \mathcal{P}(B) \subset \mathcal{P}(A \cup B)$.

(ii) Show that $\mathcal{P}(A \cup B)$ is not generally a subset of $\mathcal{P}(A) \cup \mathcal{P}(B)$.