

HOMEWORK 1

CHAPTER 1: ELEMENTARY LOGIC

1.1. Show that $\neg(\neg P) \Leftrightarrow P$ is a tautology.

1.2. Determine whether each of the following is a tautology, a contradiction, or neither one.

- (i) $(P \Rightarrow Q) \Rightarrow Q$
- (ii) $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
- (iii) $(P \Rightarrow Q) \Leftrightarrow (Q \Rightarrow P)$
- (iv) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
- (v) $(P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)$
- (vi) $P \wedge (P \Rightarrow Q) \wedge (P \Rightarrow \neg Q)$
- (vii) $(\neg Q \wedge (P \Rightarrow Q)) \Rightarrow \neg P$
- (viii) $((P \vee Q) \wedge \neg Q) \Rightarrow P$
- (ix) $((P \vee Q) \Rightarrow R) \Rightarrow (P \Rightarrow R)$

1.3. Determine whether or not each of the following is equivalent to the implication $P \Rightarrow Q$.

- (i) The converse: $Q \Rightarrow P$
- (ii) The contrapositive: $\neg Q \Rightarrow \neg P$
- (iii) The inverse: $\neg P \Rightarrow \neg Q$

1.4. Use truth tables to prove the following:

- (i) Disjunction is commutative, i.e. $P \vee Q$ is equivalent to $Q \vee P$.
- (ii) Conjunction is commutative.
- (iii) Disjunction is associative, i.e. $P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$.
- (iv) Conjunction is associative.

1.5. Show that $P \wedge (Q \vee R)$ is not equivalent to $(P \wedge Q) \vee R$.

1.6. Use truth tables for each of the following:

- (i) Show that conjunction distributes over disjunction, i.e. that $P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$.
- (ii) Show that disjunction distributes over conjunction.

1.7. Determine whether or not each of the following syllogisms is valid.

- (i) Some dogs are beagles. Snoopy is a dog. Hence Snoopy is a beagle.
- (ii) All cats are independent. Some pets are cats. Hence some pets are independent.
- (iii) No pigs fly. Porky is a pig. Hence Porky does not fly.
- (iv) No orcs are ogres. All ogres are green. Hence no orcs are green.

1.8. Negate each of the following:

- (i) Every cloud has a silver lining.
- (ii) If it's Tuesday, this must be Belgium.
- (iii) There is a light at the end of every tunnel.

1.9. Use truth tables to show that:

- (i) $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
- (ii) $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$

1.10. Use a truth table to show that $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$