Exam I Study Guide
Elementary Differential Equations
(MATH 266), Fall 2025

Location: Witmer 114

Date: Monday, September 29, 2025 from 11:15 AM - 12:05 PM

Scope of the Exam

The exam will consist of problems concerning the material we have covered thus far in the course prior to Chapter 4. The focus of your study should be given to topics that correspond to problems assigned in the homework. As a good rule of thumb, if a topic appears in the text and does not correspond to some generally analogous problem assigned in the homework, you can safely neglect it.

To score well on the exam, proficiency in the following tasks may be desirable:

(i) Identify the order of a differential equation

(ii) Identify whether a function or a parameterized family of functions is a solution to a first-order ordinary differential equation

(iii) Distinguish between implicit and explicit solutions to first-order ordinary differential equations

(iv) Discern basic properties of a solution to a first-order ordinary differential equation from the differential equation itself (i.e. draw direction fields, sketch solutions to initial value problems from direction fields, identify critical points and regions where a solution is increasing or decreasing, etc.)

(v) Identify whether a critical point of a first-order ordinary differential equation is stable, semistable, or unstable

(vi) Identify whether a first-order ordinary differential equation is written in normal form or differential form

(vii) Identify whether a first-order ordinary differential equation has a particular form. Specifically, identify whether it is separable, linear, exact, homogeneous, or Bernoulli

(viii) Solve separable, linear, exact, homogeneous, and Bernoulli first-order ordinary differential equations

(ix) Solve initial value problems involving separable, linear, exact, homogeneous, and Bernoulli first-order ordinary differential equations.

Selected Exercises from Homework Assignments

The following problems from the homework assignments are an excellent study resource for this exam:

Homework 1.1 : $1-3, 5, 8, 9$
Homework 1.2 : $1, 2$
Homework 2.1 : $1, 2$
Homework 2.2 : $1-4, 7$
Homework 2.3 : $1-4, 7-9$
Homework 2.4 : $1-3, 5, 7$
Homework 2.5 : $1, 3-5$

Selected Exercises from the Textbook

Considering exercises from the textbook may be a useful resource in preparing for the exam. In particular, you may find it helpful to review the following exercises:

Section 1.1 : $1-12, 17-20, 23-26, 33-44$
Section 1.2 : $1-14$
Section 2.1 : $1-4, 5-12$
Section 2.2 : $1-29,$
Section 2.3 : $1-24, 25-28, 31-34$
Section 2.4 : $1-26, 29-40$
Section 2.5 : $1-10, 15-20, 23-28$

You certainly do not *need* to do all of the above problems to be adequately prepared for the exam (there are far too many problems here for that). Instead, it is recommended that you just look over some of the problems on this list from sections you feel less confident about and practice by attempting whatever ones seem challenging to you. In some sections above, there are a large number of suggested problems listed. This is primarily because there are quite a few problems in the textbook which are virtually identical, so please do not take it as an indication that these sections are somehow more important.

Structure of the Exam

The exam will consist of a section involving true or false questions, a section involving some multiple choice questions, and a section where you will solve differential equations or initial value problems. The final section will be the most time consuming (since it involves actually solving differential equations), though it only consists of four problems. There will be problems involving exact equations and equations that can be solved using substitutions, as well as (either directly or in the process of solving an equation using a substitution) separable equations and linear equations.

Approaches to Solving First-Order Ordinary Differential Equations

The following may be of some help if you feel at all overwhelmed by the number of different types of first-order ordinary differential equations we have covered:

Separable : $\frac{dy}{dx} = g(x)h(y)$ **Approach** : Integrate both sides $\int \frac{1}{h(y)}dy = \int g(x)dx.$ to get a family of implicit solutions.

Linear : $a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$ **Approach** : Manipulate into the standard form $\frac{dy}{dx} + \mathrm{P}(x)y = f(x)$. Then,
$y(x) = e^{-\int \mathrm{P}(x)dx}\left(\int e^{\int \mathrm{P}(x)dx} f(x)dx\right)$
$\left(\text{but skip the +C's in } \int \mathrm{P}(x)dx\right)$

Exact : $\mathrm{M}(x,y)dx + \mathrm{N}(x,y)dy = 0$ satisfying $\frac{\partial \mathrm{M}}{\partial y} = \frac{\partial \mathrm{N}}{\partial x}$ **Approach:** Find $f(x,y)$ such that $\frac{\partial f}{\partial x} = \mathrm{M}(x,y)$ and $\frac{\partial f}{\partial y} = \mathrm{N}(x,y)$ by integrating M w.r.t. x (resp. N w.r.t. y) (and not forgetting to add a new unknown function of y (resp., x)), then differentiating w.r.t. y (resp. x) and setting this equal to N (resp. M) to find the unknown function of y (resp. x) produced by the integration.

Bernouli : $\frac{dy}{dx} + \mathrm{P}(x)y = f(x)y^n$ **Approach** : Substitute $u = y^{1-n}$ to get a linear equation.

Homogeneous : $\mathrm{M}(x,y)dx + \mathrm{N}(x,y)dy = 0$ with $\mathrm{M}(tx,ty) = t^m\mathrm{M}(x,y)$ and $\mathrm{N}(tx,ty) = t^m\mathrm{N}(x,y)$ for some m. **Approach** : Substitute $u = \frac{y}{x}$ to get a separable equation.

Other Advice

(i) Show as much of your work as you can in as neat, orderly, and coherent a manner as possible! It is hard to give partial credit if you don't give me something to work with or if you make it hard for me to tell what you were trying to do.

(ii) Don't panic if you can't evaluate an integral–the main thing I am trying to assess is your ability to solve differential equations, not your ability to compute integrals (though is is certainly still an important skill). If you get stuck, don't spend half the test focusing on a single integral for no reason–just move on and describe what you would have done using a new function to denote the result of the integration (but be clear to explain that this is what you're doing, so it doesn't seem like a new function inexplicably pops out of nowhere).